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The Effectiveness of Teaching Problem Solving Strategies According to Constructivism Theory on Mathematics Academic Achievement and Attitudes Toward Mathematics in Sixth Grade Elementary School Students

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Article Info	ABSTRACT
Article type:	Objective: The aim of the present study was to investigate the effectiveness of teaching
Research Article	problem solving strategies according to constructivism theory on mathematics academic
Article history:	achievement and attitudes toward mathematics in sixth grade elementary school students in
Received 6 Apr. 2024	Tehran.
	Methods: In a quasi-experimental study, 30 sixth grade students from elementary schools in
Received in revised form 7	Tehran were selected using convenience sampling and randomly assigned to experimental
Sep. 2024	and control groups (15 students in each group). The teacher-made academic achievement test
Accepted 17 Oct. 2024	and the attitude scale toward mathematics lessons were used to collect data. In the
Published online 01 Mar. 2025	experimental group, problem solving strategies based on the SOLO approach were
	implemented in seven stages, while the control group received conventional education.
Keywords:	Results : The findings showed that teaching problem-solving strategies based on the SOLO
Problem-solving strategies	approach can significantly increase academic achievement (T = 2.31, $p < 0.05$) and improve
according to the SOLO	attitudes towards mathematics (T = 9.25, $p < 0.05$).
approach	
approach,	Conclusions : Teaching according to Solo theory in mathematics significantly contributes to
Academic achievement in	Conclusions : Teaching according to Solo theory in mathematics significantly contributes to academic achievement and changes in students' attitudes toward mathematics. This method,
Academic achievement in mathematics,	Conclusions : Teaching according to Solo theory in mathematics significantly contributes to academic achievement and changes in students' attitudes toward mathematics. This method, by improving students' cognitive abilities and critical analysis, increases their self-confidence
Academic achievement in mathematics, Attitudes toward mathematics,	Conclusions : Teaching according to Solo theory in mathematics significantly contributes to academic achievement and changes in students' attitudes toward mathematics. This method, by improving students' cognitive abilities and critical analysis, increases their self-confidence in dealing with complex problems and improves their attitudes toward this subject.
Academic achievement in mathematics, Attitudes toward mathematics, Sixth grade students	Conclusions : Teaching according to Solo theory in mathematics significantly contributes to academic achievement and changes in students' attitudes toward mathematics. This method, by improving students' cognitive abilities and critical analysis, increases their self-confidence in dealing with complex problems and improves their attitudes toward this subject.
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Academic achievement in mathematics, Attitudes toward mathematics, <u>Sixth grade students</u> Cite this article: Kalati, Z., Rost strategies according	Conclusions : Teaching according to Solo theory in mathematics significantly contributes to academic achievement and changes in students' attitudes toward mathematics. This method, by improving students' cognitive abilities and critical analysis, increases their self-confidence in dealing with complex problems and improves their attitudes toward this subject.
Academic achievement in mathematics, Attitudes toward mathematics, Sixth grade students Cite this article: Kalati, Z., Rost strategies according mathematics in sixth g	Conclusions : Teaching according to Solo theory in mathematics significantly contributes to academic achievement and changes in students' attitudes toward mathematics. This method, by improving students' cognitive abilities and critical analysis, increases their self-confidence in dealing with complex problems and improves their attitudes toward this subject. tamy-Malkhalife, M. & Behzadi, M. H. (2025). The effectiveness of teaching problem solving to constructivism theory on mathematics academic achievement and attitudes toward grade elementary school students. <i>Iranian Journal of Educational Research</i> , 4 (1), 45-58.

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Introduction

The acquisition of mathematical knowledge is regarded as one of the most significant accomplishments within the educational framework and can be viewed as a crucial element of the academic curriculum. While numerous individuals may perceive algebra as a daunting and superfluous subject, it is imperative to acknowledge the myriad advantages associated with its mastery, as algebra is recognized as a conduit to advanced mathematical study. An incontrovertible characteristic of elementary mathematics is its primary emphasis on numerical proficiency and calculation skills. Consequently, this conventional focus has resulted in a deficiency in students' comprehension of relational concepts and the interpretation of mathematical symbols (Douglas et al., 2020). According to Herbert and Brown (1997), the process of algebraic thinking for students is characterized by the meaningful application of algebra, which involves imparting knowledge and fostering understanding, rather than merely engaging with symbols; furthermore, it allows students the opportunity to reflect upon and discuss their overarching methodologies. A substantial number of students struggle with grasping the symbols, concepts, and reasoning skills inherent in algebra (Hadi & Jawad, 2021; Sibgatullin et al., 2022). Generally, arithmetic and algebra are treated as distinct entities within the majority of mathematics curricula in educational institutions, with algebra receiving heightened attention at the onset of middle and high school (Siegler et al., 2012). This bifurcation engenders a chasm and a lack of acknowledgment between arithmetic and algebra, thereby complicating the process for students to acquire algebraic knowledge in subsequent academic years (Kargar et al., 2010). By assessing the level of students' algebraic thinking capabilities, it becomes feasible to implement suitable strategies aimed at enhancing these cognitive skills. It is essential to evaluate students' algebraic thinking proficiencies in accordance with the SOLO taxonomy, to ascertain the specific level of algebraic reasoning that students possess. Accordingly, the SOLO model was conceived in the late 1970s and aligns closely with the neo-Piagetian theoretical framework (Sudihartinih, 2019). The foundational premise of this model posits that cognitive understanding transcends Piaget's cognitive structure, encompassing individual attributes that are contingent upon the specific content and contextual factors. This model provides a systematic approach for quantifying students' cognitive abilities within the learning environment. The algebraic challenges encountered by students predominantly pertain to mathematical modeling, the interpretation of algebraic expressions, the execution of arithmetic operations, the comprehension of various interpretations of the equality sign, and the understanding of variable symbols (Rangel et al., 2016). A significant proportion of students encounter difficulties in generalizing arithmetic through the utilization of algebraic symbols (Ying et al., 2020). Cognitive transformations are necessitated in accordance with the alterations in computational processes for learners engaged in algebraic studies to facilitate the transition from one representation to another (Loibl & Leuders, 2019). Students exhibit a propensity to engage in linguistic responses rather than symbolic ones. To upgrade the challenges faced by students, educators ought to cultivate students' algebraic reasoning competencies. A plethora of studies has examined the efficacy of pedagogical strategies focused on problem-solving in relation to cognitive and emotional dimensions within the realm of mathematics education. For instance, Ocal (2017) in a study entitled "The Effect of Geogebra on Students' Conceptual and Procedural Knowledge: The Case of Applications of Derivative" demonstrates that student performance has exhibited enhancement, and the conditions established have fostered a transformation in their attitudes and superior performance. In another investigation, Fuji et al. (2003) in an article titled "Investigating Students' Understanding of Variables through Cognitive Problems: Is Understanding a Variable Difficult for Students?" concluded that a pivotal component of algebraic cognition and reasoning (algebraic thinking) is the achievement of generalization patterns, with successful generalization being contingent upon students' comprehension of the variable concept, which necessitates extensive abstract reasoning. Consequently, it is imperative that students attain this level of abstract cognition. In a study titled "Multiple Representations in Mathematics Education," Nooriafshar et al. (2017) illustrated that diverse forms of representation facilitate the transition from concrete objects and tangible manipulations to abstract thought, thereby establishing a foundation for cohesive learning. Strategies that may prove beneficial in the creation or integration of multiple representations within mathematics education are proposed, grounded in contemporary educational methodologies, particularly the utilization of information and communication technology.

<u>Nowroozi et al. (2010)</u> demonstrated in a study titled "Multiple Representations: An Important Process in Teaching and Learning Fractions" those participants in the experimental cohort exhibited enhanced performance and their conceptual understanding of fractions was significantly

advanced. The results further indicated that the experimental group's grasp of the fraction concept was more robust and enduring in comparison to that of the control group.

Yuhasriati et al. (2022) ascertained the existence of a significant interaction between multiple representation strategies and a realistic pedagogical approach, with respect to algebraic thinking proficiency. Students who employed multiple representation strategies demonstrated superior algebraic thinking proficiency compared to their counterparts engaging in conventional scientific education. Heidari and Asphary (2019) undertook a research initiative to scrutinize the comprehension of algebraic expressions among students in the seventh, eighth, and ninth grades, grounded in Sfard's object-oriented theoretical framework. For the purpose of this study, a cohort of 400 students was meticulously selected utilizing a multi-stage cluster sampling methodology, and a test devised by the researchers was developed and administered. Subsequent to the data analysis, semi-structured interviews were conducted with 15 participants to elucidate the students' comprehension. The outcomes of this investigation revealed that a predominant number of students possessed a fundamentally procedural understanding of algebraic expressions (both simple and complex), signifying that their grasp of algebraic expressions was limited to a series of algorithms and processes. When tasked with performing operations on an algebraic expression, these individuals tended to merely memorize procedural steps in a rote manner, as they lacked an understanding of the overarching structure of the algebraic expression and solely concentrated on the procedural steps involved. In the case of complex algebraic expressions, the proportion of students exhibiting a purely procedural understanding diminished relative to that of simple algebraic expressions. The insights gained from interviews with select students indicated that this reduction was not attributable to an enhancement in structural comprehension; rather, it stemmed from factors such as neglecting the distributive property, failing to grasp algebraic expressions, and lacking an understanding of the operational processes in complex algebraic expressions. The challenges and impediments encountered by students in their learning journey, particularly within the domain of algebra, warrant an examination through the lens of cognitive processing types. In instances where a student demonstrates suboptimal performance, it becomes imperative and crucial for educators to assess the components of algebraic thinking exhibited by their students. To discern issues within the domain of conceptual understanding, one methodological approach involves scrutinizing the processes by which concepts are constructed and the cognitive frameworks that learners develop to assimilate these concepts.

In this context, mathematics educators are positioned to formulate instructional strategies that facilitate the enhancement and evolution of conceptual frameworks within the cognitive architecture of students. To achieve this objective, the application of learning theories is warranted. Learning theory posits predictions regarding the mechanisms of individual learning, encompassing various tiers of the learning continuum. This investigation aims to assess the degree of algebraic reasoning among sixth-grade elementary students and to evaluate the cognitive factors that influence algebraic reasoning, in alignment with the SOLO constructivist framework; thus, the cognitive structures instantiated within the students' minds were analyzed, alongside the evaluation of conceptual comprehension and its implications for addressing algebraic reasoning tasks. Consequently, this research scrutinizes the impact of instructing problem-solving methodologies, grounded in constructivist principles, on the academic advancement of mathematics and the attitudes toward mathematics instruction among sixth-grade elementary students in Tehran.

Material and Methods

The current research methodology employed a semi-experimental post-test design incorporating a control group, utilizing the SOLO theory-informed pedagogical approach to facilitate the instruction of problem-solving strategies. The designated population for this investigation comprises sixth-grade elementary school students, totaling 1220 individuals as per institutional records. The research sample was constituted of 30 students, selected via convenience sampling techniques. Subsequently, the chosen participants were randomly allocated into two groups: an experimental group and a control group. In adherence to ethical guidelines, prior to the commencement of the study, an informed consent document was solicited and completed by the parents or guardians of the students involved in the research.

For data collection, an academic achievement assessment devised by the researcher, alongside an attitude scale toward mathematics, was employed. The final evaluative measure (post-test) was conducted using a teacher-generated end-of-year academic achievement examination. To approximate the content validity of this assessment, the researcher meticulously reviewed and

analyzed the educational resources of the textbook, assessing the number of pages and comparing it with the frequency of question utilization from each resource in relation to the aforementioned test, thereby ensuring a degree of content validity. Consequently, by quantifying the number of pages associated with the three domains of geometry, algebra, and calculus, the proportional representation of content from the respective geometry, algebra, and calculus textbooks was determined; this was subsequently compared utilizing the weighted scores of the aforementioned examination questions. The reliability of this assessment was also computed utilizing the halving method, which yielded a coefficient (r=0.78), indicating that the test possesses an acceptable and relatively high degree of reliability.

An additional research instrument employed in this study is the Attitude towards Mathematics Scale, which was developed by <u>Aiken Jr (1970)</u> utilizing the Likert methodology. This scale evaluates four emotional dimensions pertaining to mathematics instruction, specifically:

a) Enjoyment, encompassing items 1, 5, 9, 13, 17, 21

b) Motivation, encompassing items 2, 6, 10, 18, 22

c) Importance, encompassing items 3, 7, 11, 15, 19, 23

d) Fear and anxiety, encompassing items 4, 8, 12, 14, 16, 20

This scale comprises 23 items, with each item providing four response options: "I completely disagree," "I disagree," "I completely agree," and "I agree," which are assigned scores ranging from 1 to 4 in accordance with the content of each item, either in the specified order or vice versa. Ultimately, these scores are aggregated to yield a cumulative score reflecting attitudes towards mathematics. The threshold score for this scale is established at 58, where lower scores signify a negative disposition and higher scores denote a positive orientation towards mathematics.

Jamalizavareh and Nadi (2016) assessed the reliability of this scale to be 0.82 by employing the Cronbach's alpha methodology and additionally implemented it for evaluating content validity based on the perceptions of middle school mathematics educators and postgraduate mathematics scholars, which he ascertained to be valid regarding its content.

To implement the independent variable, defined as the instruction of problem-solving strategies, a structured lesson plan comprising seven sessions was utilized, during which the experimental cohort engaged in seven sessions of forty-five minutes each, organized as follows.

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First session

1 -Elucidation of problem definition, including the identification of known and unknown components

2 -Acquaintance with the categorization of problems into three distinct classifications: algebra, geometry, and arithmetic

3 -Introduction to the systematic step-by-step approach for problem-solving employing the SOLO model, which comprises four integral components:

- a) Comprehension (grasping the essence of the problem)
- b) Strategic Planning (anticipating and selecting a viable solution)
- c) Execution of the plan (applying the identified solution to arrive at the answer
- d) Evaluation and verification of the answer (assessing the outcomes)

Second session

1- Recapitulation of the objectives articulated in the preceding session

2 -Familiarization with the application of the four steps of problem-solving and the phases of the SOLO model in addressing algebraic problems

3 -Demonstration of the resolution of two algebraic problems by the instructor, incorporating the four steps of problem-solving aligned with the SOLO model

- 4 -Addressing potential challenges and responding to students' inquiries
- 5 -Assigning algebra practice as homework

Third session

1 -Review of the approach for completing homework and solving problems

- 2 -Instruction on the resolution of two additional algebraic problems, accompanied by explanations of the four steps by the instructor
- 5
- 3 -Addressing students' potential problems and responding to their questions
- 4 -Familiarization with the application of the four-step method of problem-solving and the stages of the SOLO model in tackling geometry problems
- 5 -Collaborative resolution of two sample geometry problems, with the instructor elucidating the four steps involved

Session Four

- 1 -Recapitulation of the content discussed in the preceding session
- 2 -Resolution of two sample pattern problems utilizing the same methodology as previously employed
- 3 -Addressing students' potential challenges and responding to their questions
- 4 -Assignment of exercises as homework

Session Five

- 1 -Review of the methodology for completing homework and problem-solving
- 2 -Acquaintance with the application of the four-step SOLO method for solving arithmetic problems
- 3 -Instructor-led resolution of two sample arithmetic problems, detailing the four steps involved

4 -Addressing students' potential challenges and responding to their questions

5 -Familiarization with the application of the four-step method of problem-solving and the stages of the SOLO model in solving arithmetic problems

6 -Assignment of arithmetic exercises for completion at home utilizing the SOLO method

Session Six

1 -Recapitulation of the content addressed in the preceding session

2 -Review of the methodology for completing homework and addressing potential problems

3 -Resolution of two additional arithmetic problems, accompanied by a title for the exercises

Seventh Session

Comprehensive review of the previous six sessions, alongside troubleshooting and addressing examination queries.

It is imperative to acknowledge that the exemplars of problems addressed during the instructional period were meticulously chosen from the periodic exercises contained within the textbook, while the assignments designated for homework were derived from the sample inquiries provided by the Department of Education. The duration allocated for the training sessions focused on problem-solving strategies constituted an integral component of the officially mandated attendance hours for these students at the educational institution. The educator responsible for both cohorts was identical. In collaboration with this instructor, the sample exercises administered to the experimental group were also separately conducted for the control group; however, it is pertinent to note that these exercises were exclusively conveyed to the control group without the incorporation of problem-solving skills training. To evaluate the research hypotheses, the independent groups T-test was employed utilizing the SPSS-26 statistical software.

Results

The mean and standard deviation of academic achievement and attitude toward mathematics of the control and experimental groups are presented in Table 1.

Table 1. Mean and standard deviation of academic achievement and attitude toward mathematics of the control and
experimental groups

C	Academic achievement		Attitude to mathematics	
Group	Mean	SD	Mean	SD
Control	10.85	2.02	48.53	3.71
Experimental	12.86	3.14	56.26	6.84

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According to the first hypothesis of the study, teaching problem-solving strategies increases academic achievement in mathematics, or in other words, there is a statistically significant difference between the average scores of academic achievement in mathematics of the group of students who received mathematics teaching according to Solo theory (experimental group) and the group who did not receive this training. To test the first hypothesis, a T-test for independent groups was used. The results are presented in Table 2.

Table 2. Results of the 1-test for independent groups related to the first hypothesis					
Group	Mean	SD	T value	DF	Р
Experimental	12.86	9.85	2.21	20	0.05
Control	10.85	4.11	2.31	28	0.05

Table 2. Results of the T-test for independent groups related to the first hypothesis

Since the calculated T value (2.31) is larger than the critical T value (p < 0.05). Therefore, the null hypothesis is rejected, meaning that the observed difference between the means of the two groups is significant with 95% confidence and the method of teaching mathematics according to Solo theory has had a positive effect on the academic achievement in mathematics.

According to the second hypothesis, teaching mathematics according to constructivism theory improves students' attitudes towards mathematics. In other words, there is a statistically significant difference between the mean scores of the attitude scale towards mathematics of the group of students who received mathematics teaching according to constructivism theory (experimental group) and the group who did not receive this training (control group). To test the second hypothesis, a T-test for independent groups was used. The results are presented in Table 3.

Table 5. Results of the 1-lest for independent groups related to the second hypothesis					
Group	Mean	SD	T value	DF	Р
Experimental	10.90	5.27	0.25	20	0.05
Control	6.24	0.53	9.23	28	0.05

Table 3. Results of the T-test for independent groups related to the second hypothesis

Since the calculated T value (9.25) is significant (p < 0.05), the null hypothesis is rejected, meaning that it can be said with 95% confidence that teaching mathematics according to Solo's theory has had a positive effect on students' attitudes towards mathematics.

Discussion

The objective of the current research was to examine the efficacy of imparting problem-solving strategies in accordance with constructivist theory on the academic advancement in mathematics and the attitudes toward mathematics among sixth-grade elementary school pupils in Tehran. The results indicated that the pedagogical approach to mathematics informed by Solo theory exerted a beneficial influence on both the academic advancement in mathematics and the students' attitudes towards the subject. These results align with the findings of <u>Nowroozi et al. (2010)</u>, <u>Heidari and Asghary (2019)</u>, and <u>Nooriafshar et al. (2017)</u>.

The pedagogical approach to mathematics informed by constructivist theory (Solo) has a favorable impact on both academic progress and students' attitudes towards mathematics. To gain a more precise understanding of these effects, it is imperative to first become acquainted with the concept of Solo theory and its application within educational practices. Solo theory (Structure of Observed Learning Outcome) represents an educational framework that delineates and assesses the learning processes and cognitive evolution of students. This theory specifically emphasizes the progression of understanding concerning increasingly intricate mathematical concepts and posits that learners operate at varying levels of cognitive sophistication. These levels encompass:

1. Pre-structural level: Students exhibit an inability to comprehend the concept and retain only fragmented information.

2. Mono-structural level: Students grasp one or more facets of the concept but have yet to establish comprehensive connections among the various components.

3. Poly-structural level: At this stage, students can correlate multiple facets of a concept.

4. Convergent level: Students achieve the ability to recognize interrelations and attain a more holistic comprehension of the problem as an entirety.

5. Abstract level: At this stage, students can address complex and abstract issues utilizing abstract concepts and methodologies.

Employing Solo's theoretical framework in the pedagogy of mathematics entails that educators systematically and intentionally facilitate the progression of learners from foundational (pre-constructive) stages to higher-order (abstract) stages. Within this pedagogical model, curricular design is tailored to reflect the unique knowledge and competencies of each student, emphasizing profound comprehension of mathematical principles rather than mere rote memorization. Through

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the application of this instructional method, learners develop the capacity to scrutinize and resolve increasingly intricate problems, alongside the acquisition of fundamental mathematical concepts. Given that this methodology perpetually steers students toward elevated echelons of cognitive engagement, there is a marked enhancement in their academic performance in mathematics. Furthermore, students attain proficiency in tackling not only elementary problems but also more demanding and sophisticated challenges, thereby bolstering their self-efficacy in the realm of mathematics. Within the context of Solo's theory, the emphasis on the educational process, rather than solely on conclusive outcomes, engenders a transformation in students' perceptions of mathematics. As students cultivate a more profound understanding of mathematical concepts and recognize their capacity to approach problem-solving through diverse methodologies, their disposition toward the subject matter becomes increasingly positive. This pedagogical approach fosters a sense of fulfillment and accomplishment among students, particularly when they realize that they can attain success through their own devised strategies and techniques.

Solo's theoretical framework underscores the notion that learning transpires sequentially across stages, advocating for instructional practices that align with these developmental phases. This approach prioritizes the active engagement of students, thereby ensuring their participation throughout the learning continuum and fostering a tangible understanding of mathematical constructs. In contrast to conventional instructional methodologies that may predominantly emphasize the dissemination of detailed information, this educational strategy equips students with a holistic perspective of mathematical concepts, enabling them to adeptly navigate and resolve more complex problems. Ultimately, the implementation of Solo's theory in the instruction of mathematics significantly enhances academic performance and positively influences students' attitudes toward the discipline. This pedagogical approach augments students' self-assurance in addressing intricate issues by refining their cognitive faculties and critical thinking skills, thereby improving their overall disposition toward the subject matter.

A recommendation for subsequent inquiry is to investigate the impact of these methodologies across various educational tiers or among students with disparate abilities to derive more broadly applicable conclusions. Additionally, the implications of these instructional strategies could be explored within the context of other academic disciplines. Another avenue for research is to utilize larger and more heterogeneous samples across diverse geographic regions to facilitate a more comprehensive analysis of the findings. With regard to limitations, one potential challenge may be the constrained duration of the study and the resultant short-term effects, necessitating further investigation to evaluate the enduring implications. Furthermore, inconsistencies in the precise execution of the teaching methodology across all classroom settings may represent one of the constraints encountered.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Ethics statement

The studies involving human participants were reviewed and approved by ethics committee of Islamic Azad University.

Author contributions

All authors contributed to the study conception and design, material preparation, data collection and analysis. The author contributed to the article and approved the submitted version.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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