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Unpacking the Cognitive Architecture of Creative Mathematical Reasoning: A Think-Aloud Protocol Analysis of Student Problem-Solving Strategies

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ABSTRACT

Objective: This study addresses this gap by employing a think-aloud protocol to investigate the cognitive strategies and processes characteristic of CMR in eighth-grade students. Understanding these mechanisms is vital for developing effective pedagogical approaches that foster deeper mathematical understanding and innovation.

Methods: This research utilized a qualitative approach, implementing a think-aloud protocol with six eighth-grade students as they solved number-pattern problems. Participants verbalized their thought processes in real-time, providing rich data for analysis.

Results: The analysis identified five key cognitive strategies students engaged in: visual-spatial analysis, combined deductive-inductive reasoning, flexible use of multiple representations, metacognitive self-monitoring, and iterative reconstruction. Furthermore, the study delineated three primary phases of CMR: exploration and pattern recognition, abstraction and formulation, and verification and generalization. Distinct differences emerged between students who successfully exhibited CMR and those who struggled. Successful problem-solvers demonstrated more systematic visualization, robust self-monitoring, and greater flexibility in switching between strategies. Conversely, less successful students often exhibited limitations in these areas, resorting to less adaptive approaches.

Conclusions: The findings indicate that CMR is not an innate talent but a set of teachable cognitive skills. The study highlights the efficacy of explicit strategy instruction, scaffolding metacognitive awareness, and encouraging reflective practices in cultivating CMR. These insights have significant implications for educational reform, suggesting that curricula and teaching methods should be adapted to prioritize the development of these strategic and metacognitive components. By fostering CMR, educators can equip students with the creative problem-solving abilities necessary for success in increasingly complex mathematical domains and future academic pursuits.

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Introduction

One of the fundamental challenges in mathematics education worldwide is the deep gap between knowing mathematical algorithms and formulas on one hand, and deep conceptual understanding and the ability to reason creatively on the other (Hiebert & Lefevre, 1986; Schoenfeld, 1985). This problem, rooted in traditional approaches to mathematics teaching, has been extensively discussed in the research literature. Numerous studies have shown that many students, even after years of formal mathematics education, are unable to solve non-routine problems or transfer their knowledge to new situations (Lithner, 2008; National Council of Teachers of Mathematics, 2000). In recent decades, mathematics education researchers have increasingly paid attention to the importance of mathematical reasoning as a fundamental competency. Mathematical reasoning goes beyond mechanical calculations and application of predetermined formulas and includes the ability to construct and evaluate logical proofs, discover patterns, generalize findings, and use inherent mathematical properties to solve problems (Tall, 2004). In this regard, Lithner (2008) made an important distinction between two types of mathematical reasoning: imitative reasoning and creative reasoning. Creative Mathematically Founded Reasoning (CMR) requires the construction of new reasoning based on deep understanding of mathematical concepts and properties.

According to Lithner's (2008) framework, CMR has three fundamental characteristics: novelty (construction of new reasoning), plausibility (support with logical proof), and mathematical foundation (reliance on inherent mathematical properties). Despite growing evidence about the benefits of CMR, precise understanding of the cognitive processes that students employ while using this type of reasoning remains limited (Lithner, 2003; Sfard, 1991).

The Think-Aloud Protocol, introduced by Ericsson and Simon (1984), offers a powerful method for accessing ongoing cognitive processes. This study aims to address the knowledge gap by providing an in-depth analysis of students' cognitive processes during CMR using think-aloud protocol.

Material and Methods

Research Design

This research was conducted using a qualitative approach with a case study design, appropriate for achieving deep and detailed understanding of cognitive processes (Stake, 1995). Qualitative case studies allow researchers to examine phenomena in their natural context with complete detail.

Participants

Participants were six eighth-grade students (three boys and three girls) aged 13-14 years, selected through purposive sampling from an urban school. Three students with high performance (scores 18-20/20) and three students with average performance (scores 10-12/20) were selected to enable comparison of their reasoning approaches. All participants and their parents provided informed consent.

Data Collection

Data collection was conducted through individual think-aloud sessions. Each student was presented with carefully designed number pattern problems and asked to verbalize all thoughts during problem-solving. Sessions were video and audio recorded, with an average duration of 45-60 minutes per student. Prior to data collection, students participated in brief training sessions to become familiar with the think-aloud technique.

Data Analysis

Data were analyzed through qualitative content analysis using Grounded Theory approach (Strauss & Corbin, 1998), consisting of three stages: open coding (identifying initial concepts), axial coding (establishing relationships between categories), and selective coding (integrating categories into a coherent theoretical framework). All recordings were transcribed verbatim and analyzed systematically to identify patterns in students' cognitive processes.

Results

Five Main Cognitive Strategies in Creative Mathematically Founded Reasoning

Analysis of think-aloud protocols revealed five main cognitive strategies that students employed during CMR. These strategies represent distinct but interconnected approaches to mathematical problem-solving, each contributing uniquely to successful creative reasoning. The following sections provide detailed explanations of each strategy, including concrete examples from the data and their theoretical significance.

Visual-Spatial Analysis: Constructing External Representations: Visual-spatial analysis emerged as a foundational cognitive strategy in which students systematically used visual and spatial representations to understand problem structure and discover underlying mathematical relationships. This strategy goes beyond simple diagram drawing; it involves the deliberate construction and manipulation of external representations to support internal cognitive processing. Successful students demonstrated sophisticated visual-spatial analysis by creating multiple types of representations. They drew diagrams showing the geometric structure of patterns, constructed tables to organize numerical data systematically, used arrows and lines to highlight relationships between elements, and employed color coding to distinguish different components of patterns. For example, when solving a number pattern problem, Student A (high performer) began by sketching the first three terms visually, then organized these in a table showing term number and term value, and finally used colored markers to identify which parts of the pattern changed and which remained constant.

The cognitive function of visual-spatial analysis is multifaceted. First, external representations reduce cognitive load by offloading information from working memory to paper, allowing students to focus on pattern analysis rather than remembering specific values. Second, visual representations make abstract relationships concrete and perceptible, enabling pattern recognition that might be difficult through purely symbolic manipulation. Third, diagrams and tables provide a structured framework that guides systematic exploration and hypothesis testing.

In contrast, unsuccessful students showed limited use of visual-spatial analysis. When they did create visual representations, these tended to be incomplete, unsystematic, or abandoned before yielding insights. For instance, Student D (average performer) began drawing a diagram but stopped after the first term, commenting that it was "taking too long" and attempted to solve the problem mentally instead. This premature abandonment of visual strategies often led to errors and incomplete solutions.

Combined Deductive-Inductive Reasoning: Bidirectional Thinking: The second major strategy involved the flexible integration of both inductive reasoning (generalizing from specific cases to broader patterns) and deductive reasoning (applying general principles to specific situations). Successful students did not rely exclusively on either approach but moved fluidly between them, using each mode of reasoning to complement and verify the other.

The typical pattern observed was that students began with inductive reasoning, examining several specific cases to identify regularities. For example, Student B calculated the values for the first five terms of a pattern (3, 5, 7, 9, 11) and noticed that "each number is 2 more than the previous one." This inductive observation then led to the hypothesis that the general rule might be "add 2 each time" or "2n + 1" where n is the term number.

However, successful students did not stop at inductive generalization. They then shifted to deductive reasoning to test their hypotheses. Student B, after proposing the formula "2n + 1," systematically verified it by substituting different values of n and checking whether the formula produced the correct term values. When the formula worked for all tested cases, the student then attempted to explain why the formula made sense based on the underlying structure of the pattern: "The pattern starts at 1 and adds 2 groups of n, so it's 1 + 2n."

This bidirectional movement between inductive and deductive reasoning served multiple cognitive functions. Induction provided the creative spark—the generation of hypotheses and conjectures based on observed patterns. Deduction provided verification and justification, ensuring that proposed generalizations were logically sound and mathematically grounded. The combination of both approaches exemplifies the essence of CMR: reasoning that is both novel (through inductive discovery) and mathematically founded (through deductive justification).

Unsuccessful students, by contrast, often relied heavily on induction without sufficient deductive verification. They would propose a rule based on limited examples but fail to test it thoroughly or explain why it should work. This led to overgeneralizations and errors when their inductively derived rules failed to account for all cases.

Flexible Use of Multiple Representations: Translating Between Mathematical Languages: The third cognitive strategy involved the ability to flexibly move between different mathematical representations—visual (diagrams and geometric figures), numerical (tables and calculations), verbal (descriptions and explanations), and symbolic (algebraic formulas and equations). This representational fluency allowed successful students to view problems from multiple perspectives and to verify solutions through different approaches.

Each representational system offers unique affordances for mathematical reasoning. Visual representations make spatial relationships explicit and support pattern recognition. Numerical representations allow for precise calculations and hypothesis testing through specific examples. Verbal representations facilitate conceptual understanding and communication of reasoning. Symbolic representations enable generalization and formal mathematical manipulation. Successful students leveraged the strengths of each representation type.

Student C exemplified this strategy when solving a complex pattern problem. She began with a visual representation, drawing the first four terms as geometric shapes. She then translated this into a numerical table showing how the number of elements in each term increased. Next, she verbally described the pattern: "Each term has 4 more squares than the previous term, and the first term has 3 squares." Finally, she translated this verbal description into an algebraic formula: "4n - 1 where n is the term number." Crucially, she then moved back to the visual representation to verify that her formula made sense geometrically.

This flexible representational switching served several cognitive functions. First, it provided multiple entry points into the problem—if one representation proved difficult, students could try another approach. Second, it enabled cross-verification—solutions derived in one representation could be checked against others, increasing confidence in correctness. Third, it deepened conceptual understanding by revealing connections between different aspects of mathematical concepts.

Unsuccessful students typically relied on a single preferred representation, usually numerical or visual, without attempting to translate between representation types. This limitation restricted their problem-solving flexibility and made it difficult for them to verify solutions or recover from errors. When their preferred representation did not readily yield a solution, they often became stuck rather than trying an alternative approach.

Metacognitive Self-Monitoring: Thinking About Thinking: Metacognitive self-monitoring emerged as perhaps the most distinctive difference between successful and unsuccessful students. This strategy involves continuous awareness and regulation of one's own cognitive processes, including planning, monitoring progress, evaluating solutions, and adjusting strategies when necessary. Successful students demonstrated active metacognition throughout their problem-solving process.

Evidence of metacognitive self-monitoring appeared in students' verbal protocols through various types of self-directed statements. Planning statements reflected strategic thinking about problem-solving approaches: "I should probably draw this out first to see the pattern clearly" (Student A). Monitoring statements showed ongoing awareness of progress and understanding: "Okay, this is working for the first three terms, but I need to check if it works for all of them" (Student B). Evaluation statements demonstrated critical assessment of solutions: "Wait, this doesn't make sense—let me recalculate" (Student C). Regulation statements indicated strategic adjustments: "This method isn't working. Maybe I should try organizing the information in a table instead" (Student A).

Successful students also engaged in self-questioning, asking themselves probing questions to guide their reasoning: "Does this make sense? Is there a pattern here? Why does this work? What if I tried a different approach? How can I be sure this is correct?" This internal dialogue served to maintain critical thinking and prevent premature acceptance of unverified solutions.

The cognitive function of metacognitive self-monitoring is to provide executive control over the problem-solving process. Rather than proceeding mechanically through solution attempts, metacognitive students actively steered their cognitive processes, catching errors early, avoiding unproductive paths, and selecting effective strategies. This self-regulation distinguished creative, thoughtful reasoning from mechanical, algorithmic approaches.

Unsuccessful students showed remarkably little metacognitive self-monitoring. Their protocols contained few evaluative or monitoring statements. They tended to proceed with their first approach without questioning its effectiveness, failed to notice errors or inconsistencies in their reasoning, and rarely adjusted their strategies when encountering difficulties. This lack of metacognition often resulted in perseveration on ineffective approaches or premature abandonment of problems when initial attempts failed.

Iterative Reconstruction and Verification: The Refinement Cycle: The fifth cognitive strategy involved a cyclical process of proposing solutions, testing them rigorously, identifying inadequacies, and reconstructing improved versions. Successful students did not settle for their first solution but engaged in multiple cycles of refinement, treating initial solutions as provisional hypotheses to be verified and improved rather than final answers.

This iterative process typically followed a recognizable pattern. First, students generated an initial solution or hypothesis based on their preliminary analysis. Second, they systematically tested this solution against multiple cases, including edge cases and examples beyond those initially examined. Third, they checked the solution for consistency with mathematical properties and problem constraints. Fourth, if verification revealed problems or if they identified ways to improve their reasoning, they reconstructed their solution from a different perspective. This cycle continued until students achieved a solution that satisfied their verification criteria and made sense from multiple viewpoints.

Student A provided a clear example of iterative reconstruction when working on a pattern problem. His initial solution was a recursive formula describing how each term related to the previous term. However, upon reflection, he recognized that this formula, while correct, was not in the closed form typically expected. He then reconstructed his solution, deriving an explicit formula that directly calculated any term from its position. Even after arriving at this formula, he continued to verify it by testing additional cases and explaining why the formula made mathematical sense given the problem structure.

The cognitive value of iterative reconstruction lies in its support for both error correction and deepening understanding. By repeatedly examining solutions from different angles and subjecting them to rigorous testing, students caught errors that might otherwise go unnoticed. More importantly, the process of reconstructing reasoning from multiple perspectives led to deeper, more robust understanding of the underlying mathematical concepts and relationships.

Unsuccessful students typically engaged in minimal verification and rarely reconstructed their solutions. Once they arrived at an answer that seemed plausible, they accepted it without thorough testing or consideration of alternative approaches. When asked to verify their solutions, they often simply re-executed the same procedure rather than checking from a different perspective. This lack of iterative refinement meant that errors went undetected and understanding remained superficial.

Three Key Phases in the CMR Process

Analysis revealed that the CMR process typically progresses through three distinct phases, each characterized by specific cognitive activities and goals. While these phases are presented sequentially, students often cycled between them, returning to earlier phases when verification revealed problems or when new insights suggested alternative approaches.

Phase 1: Exploration and Pattern Recognition: Students began by exploring the problem through examining specific cases, identifying regularities and patterns, and forming initial hypotheses. This phase involved extensive use of visual representations and experimentation with different examples. The primary cognitive goal was to develop an intuitive sense of the problem structure and to identify promising patterns that might lead to general solutions.

Phase 2: Abstraction and Formulation: After discovering patterns, students attempted to abstract and generalize their findings, formulating general rules or relationships. This phase required moving from specific cases to general principles and often involved translating visual patterns into symbolic or algebraic expressions. The cognitive challenge was to capture the essential structure of the pattern in a form that could be applied to any case, not just the specific examples examined.

Phase 3: Verification and Generalization: In the final phase, students verified their formulated rules by testing them against new cases, checking consistency with mathematical properties, and attempting to prove or justify their solutions. Successful students spent significant time in this phase, while unsuccessful students often skipped or rushed through it. The cognitive goal was to ensure that proposed solutions were not only correct but also mathematically justified and generalizable.

Contrasting Profiles: Successful vs. Unsuccessful Students

Analysis revealed clear and consistent differences between successful and unsuccessful students across multiple dimensions of cognitive processing, metacognitive awareness, and problem-solving behavior. These differences were not merely quantitative but reflected fundamentally different approaches to mathematical reasoning.

Successful students used systematic visual representations, maintained continuous self-monitoring throughout the problem-solving process, revised approaches when encountering errors, flexibly moved between multiple representations, and showed persistence when facing challenges. They viewed errors as opportunities for learning and refinement rather than failures. Their protocols were characterized by strategic thinking, frequent self-questioning, and iterative refinement of solutions.

In contrast, unsuccessful students showed limited visual representation use, rare self-monitoring, tendency to give up when facing difficulties, reliance on single representations without exploring alternatives, and low confidence in problem-solving abilities. They often proceeded mechanically through solution attempts without strategic planning or critical evaluation. When their initial approaches failed, they tended to abandon problems rather than try alternative strategies. Their lack of metacognitive awareness meant they often failed to recognize errors or assess the adequacy of their solutions.

Discussion

Theoretical Implications: Advancing Models of Mathematical Reasoning

This research makes several important theoretical contributions to our understanding of creative mathematical reasoning. First, it extends Lithner's (2008) framework by providing detailed, empirically-grounded description of the cognitive and metacognitive processes underlying CMR. While Lithner's framework defines CMR through its characteristics (novelty, plausibility, mathematical foundation), our study reveals the specific cognitive strategies and processes that enable students to achieve these characteristics.

The identification of five main cognitive strategies—visual-spatial analysis, combined deductive-inductive reasoning, flexible use of multiple representations, metacognitive self-monitoring, and iterative reconstruction—offers a comprehensive model for understanding how students engage in creative mathematical reasoning. These strategies are not isolated skills but form an integrated system in which each strategy supports and enhances the others. For example, visual-spatial analysis provides concrete material for inductive reasoning, while metacognitive self-monitoring guides the selection and coordination of different strategies.

Our findings particularly emphasize the central role of metacognition in successful mathematical reasoning, a dimension that has received insufficient attention in previous CMR research. The stark difference in metacognitive self-monitoring between successful and unsuccessful students suggests that metacognition may be the executive function that coordinates other cognitive strategies, determines when to switch between representations, and guides the iterative refinement process. This positions metacognition not as one strategy among many but as a higher-order cognitive capacity that orchestrates the entire reasoning process.

The three-phase model (exploration-abstraction-verification) provides a temporal structure for understanding how creative reasoning unfolds over time. This model aligns with and extends existing theories of mathematical problem-solving, such as Polya's (1945) four-stage model and Schoenfeld's (1985) framework, by specifically focusing on how students construct new reasoning rather than applying learned procedures. Our model highlights that verification and generalization are not mere final checks but integral components of the creative reasoning process, during which understanding deepens and solutions are refined.

Furthermore, our findings support and extend Duval's (2006) theory of semiotic representation systems in mathematics education. The flexible use of multiple representations demonstrated by successful students confirms that representational fluency is not merely convenient but essential for deep mathematical understanding. Students who could translate between visual, numerical, verbal, and symbolic representations demonstrated more robust and flexible reasoning than those who relied on single representations.

Comparison with Previous Literature: Situating the Findings

Our findings align with and extend previous research on mathematical reasoning in several important ways. The importance of visual-spatial analysis found in this study is consistent with research by Presmeg (2006) and Arcavi (2003), who have argued for the critical role of visualization in mathematical thinking. However, our study goes beyond simply identifying visualization as important by showing how successful students systematically use visual representations in coordination with other strategies.

The combined use of inductive and deductive reasoning observed in this study resonates with findings from Stylianides and Stylianides (2009) regarding the importance of both empirical and deductive reasoning in mathematics learning. Our data add nuance by showing that successful students do not simply use both types of reasoning but strategically move between them, using induction for discovery and deduction for verification and justification.

The centrality of metacognition found in our study strongly supports the extensive literature on the role of metacognition in mathematical problem-solving (Schoenfeld, 1992; Garofalo & Lester, 1985). However, while much previous research has focused on metacognition in the context of applying known procedures, our study demonstrates that metacognition is equally if not more critical when students engage in creative, novel reasoning. The metacognitive self-monitoring we observed goes beyond simply checking calculations to include ongoing evaluation of the reasonableness, mathematical validity, and completeness of reasoning approaches.

The differences between successful and unsuccessful students found in this study parallel findings from comparative studies of expert-novice problem-solving (Chi, Feltovich, & Glaser, 1981; Larkin et al., 1980). Like experts in other domains, successful mathematics students in our study demonstrated deeper analysis of problem structure, more strategic planning, greater flexibility in approach, and more thorough verification. This suggests that mathematical reasoning expertise, like expertise in other domains, is characterized by integrated systems of knowledge, strategies, and metacognitive skills rather than isolated competencies.

Practical Implications for Mathematics Education: From Research to Classroom

The findings have several important and actionable implications for mathematics education practice. Most fundamentally, they demonstrate that mathematical reasoning is not an innate talent possessed by some students and lacking in others, but rather a complex set of cognitive and metacognitive strategies that can be explicitly taught and developed. This shifts the educational challenge from identifying students who "can" do mathematics to designing instruction that develops reasoning abilities in all students.

First, instruction should explicitly teach the five identified cognitive strategies rather than assuming students will discover them independently. Teachers can model these strategies through think-aloud demonstrations, provide scaffolded practice opportunities, and give explicit feedback on students' use of these strategies. For example, teachers might demonstrate how to create systematic visual representations, model the process of testing hypotheses through both inductive and deductive reasoning, show how to translate between different mathematical representations, verbalize metacognitive monitoring during problem-solving, and demonstrate iterative refinement of solutions.

Second, mathematics curriculum and instruction should emphasize process and reasoning rather than only final answers. Traditional mathematics assessment focuses almost exclusively on whether students arrive at correct answers, giving little credit for reasoning process. Our findings suggest that assessment practices should value and evaluate the quality of reasoning, use of appropriate strategies, metacognitive monitoring, and thoroughness of verification. This might include requiring students to explain their reasoning in words and diagrams, show multiple solution approaches, justify why their solutions are correct, and reflect on their problem-solving process.

Third, regular opportunities for think-aloud practice and metacognitive reflection should be integrated into mathematics instruction. The think-aloud protocol itself proved to be not merely a research method but a potentially powerful pedagogical tool. When students verbalize their thinking, they become more aware of their cognitive processes, can identify gaps or errors in their reasoning, and develop metacognitive skills. Teachers might incorporate regular "think-aloud partnerships" where students solve problems while verbalizing their thinking to a peer, or use written reflection prompts that encourage metacognitive awareness.

Fourth, task design is critical. Mathematics instruction should include rich, non-routine problems that require creative reasoning rather than memorized procedures. These tasks should be at an appropriate level of difficulty—challenging enough to require creative thinking but not so difficult that students become discouraged. Tasks should also explicitly invite multiple representations and solution approaches, rewarding students who explore different methods rather than only finding a single correct answer quickly.

Fifth, scaffolding and gradual release of responsibility should guide instruction. Initially, teachers might provide substantial support by suggesting which strategies to use, guiding students through the three phases of creative reasoning, and helping students monitor their thinking. Over time, this support should be systematically reduced as students internalize these strategies and processes, eventually enabling students to engage in creative reasoning independently.

Professional Development Implications: Supporting Teachers

Implementing the recommendations above requires significant changes in teaching practice, which in turn requires comprehensive professional development for teachers. Many teachers themselves learned mathematics through traditional, procedural approaches and may lack experience with creative reasoning. Professional development programs should help teachers understand what creative mathematical reasoning looks like, develop their own creative reasoning abilities, learn how to recognize and respond to students' reasoning, design and select tasks that promote creative reasoning, and facilitate classroom discussions that support reasoning development.

Effective professional development should include opportunities for teachers to engage in creative reasoning themselves, analyze student thinking using frameworks like the one developed in this study, practice facilitating reasoning-focused instruction, and receive ongoing support and feedback as they implement new approaches in their classrooms. Video examples of successful creative reasoning, like those captured in think-aloud protocols, could be powerful professional development tools.

Connections to 21st Century Skills: Broader Educational Goals

The cognitive strategies identified in this study extend beyond mathematics to align with broader educational goals for 21st century skills. Critical thinking, problem-solving, creativity, and metacognition are widely recognized as essential competencies for success in contemporary society (Partnership for 21st Century Learning, 2019). The strategies we identified—particularly metacognitive self-monitoring, flexible use of multiple representations, and iterative refinement—are valuable across disciplines and domains.

This suggests that developing creative mathematical reasoning serves dual purposes: it improves mathematical understanding and performance while simultaneously developing transferable cognitive skills. Mathematics education, when properly focused on reasoning rather than procedure, becomes a powerful context for developing general thinking skills that students can apply throughout their educational and professional careers.

Limitations and Methodological Considerations

This study has several limitations that should be acknowledged and that point toward important areas for future research. First, the small sample size (six students) limits the generalizability of findings. While case study methodology is appropriate for developing deep understanding of cognitive processes, larger-scale studies are needed to determine how widely these findings apply across different student populations, cultural contexts, and educational systems.

Second, the focus on number pattern problems means that findings may not fully transfer to other mathematical domains such as geometry, algebra, or calculus. Different mathematical content areas may require or afford different cognitive strategies. Future research should examine creative reasoning across diverse mathematical content to identify both domain-general and domain-specific strategies.

Third, the think-aloud protocol, while providing rich data on cognitive processes, may affect natural problem-solving. Verbalizing thoughts requires cognitive resources and may alter the problem-solving process itself. Some students may find verbalizing easier than others, potentially affecting performance. However, research on think-aloud methodology suggests that when properly implemented, its effects on problem-solving are minimal (Ericsson & Simon, 1984).

Fourth, this study provides a snapshot of students' reasoning at a single point in time. It does not address how creative reasoning develops over time or how students move from unsuccessful to successful reasoning patterns. Longitudinal research is needed to understand the developmental trajectory of creative mathematical reasoning and to identify critical transitions in reasoning development.

Fifth, the study did not investigate the influence of prior instruction, home environment, or other contextual factors that may shape students' reasoning approaches. Future research should examine how instructional practices, curriculum materials, assessment systems, and cultural factors influence the development of creative reasoning.

Conclusion

This research explored cognitive processes in Creative Mathematically Founded Reasoning through detailed analysis of think-aloud protocols from eighth-grade students. The identification of five main cognitive strategies—visual-spatial analysis, combined deductive-inductive reasoning, flexible use of multiple representations, metacognitive self-monitoring, and iterative reconstruction—provides a comprehensive framework for understanding CMR. The three-phase model (exploration-abstraction-verification) describes the temporal structure of the creative reasoning process. The clear differences between successful and unsuccessful students demonstrate that mathematical reasoning ability is teachable rather than innate.

These findings have important implications for transforming mathematics education from a system that sorts students based on presumed mathematical ability to one that systematically develops reasoning abilities in all students. The key is recognizing that the cognitive and metacognitive strategies underlying creative reasoning can be explicitly taught through appropriate instructional approaches. With instruction that focuses on explicit strategy teaching, scaffolding, process emphasis, and regular reflection opportunities, all students' mathematical reasoning abilities can be systematically improved.

Realizing this vision requires sustained effort from multiple stakeholders. Researchers must continue investigating how creative reasoning develops and how instruction can most effectively support it. Teacher educators must prepare teachers to facilitate reasoning-focused instruction. Curriculum developers must design materials that promote creative reasoning. Assessment specialists must develop methods to evaluate reasoning quality. Policymakers must support educational systems that value reasoning over procedure memorization.

The potential rewards of this effort are substantial. Mathematics education that develops creative reasoning abilities serves not only to improve mathematical achievement but also to prepare students with critical thinking, problem-solving, and metacognitive skills essential for success in an increasingly complex world. This makes the effort not merely worthwhile but essential for 21st century education.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Ethics statement

The studies involving human participants were reviewed and approved by the ethics committee of Islamic Azad University. The patients/participants provided their written informed consent to participate in this study.

Author contributions

All authors contributed to the study conception and design, material preparation, data collection, and analysis. All authors contributed to the article and approved the submitted version.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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